May 2005

Avoiding the gauge heirarchy problem with see-sawed neutrino masses

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We show that the see-saw neutrino mass mechanism can coexist naturally with an extended gauge symmetry (i.e. without any gauge heirarchy problem) provided that the gauge symmetry contains gauged lepton number differences. The simplest such 'natural' see-saw models are constructed and their implications for neutrino anomalies discussed.

PACS numbers:

There is compelling evidence for non-zero neutrino masses arising from the solar and atmospheric neutrino anomalies and various terrestrial experiments[1]. A simple explanation of the required small neutrino masses, $m_\nu \stackrel{<}{\sim} eV$, is provided through the see-saw mechanism[2], which relies on a much larger Majorana right-handed neutrino mass scale $M \stackrel{>}{\sim} 10^6$ GeV. Such a large scale would be problematic if it were generated by the vacuum expectation value of a Higgs boson, since this would lead to fine tuning in the Higgs potential, both at the tree-level and radiatively[3]. However, this may not necessarily occur since the right-handed neutrinos are electroweak gauge singlets, which means that their mass scale might arise from bare mass terms:

$$\mathcal{L} = M_R^{ij}\bar{\nu}_{iR}(\nu_{jR})^c \tag{1}$$

If this were the case then there is no fine tuning problem in the theory at the classical or tree-level. This would greatly alleviate the gauge heirarchy problem.

Radiative corrections to the Higgs mass still arise which are of order[4]:

$$\delta\mu^2 \approx \frac{\lambda^2}{(2\pi)^2}M_R^2 log(\Lambda/M_R) \tag{2}$$

where λ is the Higgs charged lepton Yukawa coupling (and Λ a momentum cutoff). If we assume that $\delta\mu^2 \stackrel{<}{\sim} TeV$ then $M_R \stackrel{<}{\sim} 10^7 - 10^8$ GeV for neutrino masses of order 10^{-1} eV[4]. This suggests only a relatively low scale for M_R is 'natural'. Nevertheless, such a relatively low scale for M_R is still quite interesting making such 'natural' see-saw models worthy of study.

Our notation for the standard model quarks and leptons under the standard gauge symmetry, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, is

$$\begin{aligned} f_{iL} &\sim (1,2,-1),\ e_{iR} \sim (1,1,-2) \\ Q_{iL} &\sim (3,2,1/3),\ u_{iR} \sim (3,1,4/3),\ d_{iR} \sim (3,1,-2/3) \end{aligned} \tag{3}$$

where $i = 1,2,3$ is the generation index.

Our assumptions are

- Three right-handed neutrinos exist, which transform as $\nu_{iR} \sim (1,1,0)$ under the standard model gauge symmetry.
- The right-handed neutrinos transform non-trivially under some extended gauge symmetry.
- We assume see-saw neutrino mass mechanism, with the right-handed neutrino mass scale ($\sim 10^7$ GeV) set by bare masses. As explained above, this is reasonable in order to avoid the gauge heirarchy problem.

Our task now is to derive the implications of the above assumptions.

All gauge symmetries contain a $U(1)_{L'}$ subgroup and the assumption that the right-handed neutrinos transform non-trivially means that at least one of the ν_{iR} has a non-zero L' charge. Define L_e, L_μ, L_τ such that $L_e f_{1L} =$

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f_{1L}, $L_e e_{1R} = e_{1R}$ and $L_e F = 0$ for $F \neq f_{1L}, e_{1R}$. L_μ, L_τ are similarly defined in the obvious way. Then the most general $U(1)$ that is a (classical) symmetry of the standard model Lagrangian terms is generated by:

$$L' = aL_e + bL_\mu + cL_\tau + \alpha L_{\nu_{1R}} + \beta L_{\nu_{2R}} + \gamma L_{\nu_{3R}} + \epsilon B + \omega Y \tag{4}$$

where $a, b, c, \alpha, \beta, \gamma, \epsilon, \omega$ are arbitrary parameters and $L_{\nu_{1R}}$ is the right-handed ν_{1R} number (i.e. $L_{\nu_{1R}}\nu_{1R} = \nu_{1R}$ and $L_{\nu_{1R}}F = 0$ for $F \neq \nu_{1R}$). $L_{\nu_{2R}}$ and $L_{\nu_{3R}}$ are similarly defined in the obvious way. B is the baryon number (defined in the usual way with all quarks having charge 1/3).

In order to have light neutrinos without also having a gauge heirarchy problem, we require all three bare Majorana right-handed neutrino mass eigenvalues to be non-zero. Define the right-handed neutrino bare mass matrix as follows:

$$(\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R}) \begin{pmatrix} A & X & Y \\ X & B & Z \\ Y & Z & C \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix} \tag{5}$$

The bare masses must be $U(1)_{L'}$ invaraint. If the L' charge of at least one of the ν_{iR} is non-zero, then without loss of generality, we can assume that $L'\nu_{1R} \neq 0$, which means that $A = 0$. For ν_{1R} to have a bare mass, we require either X or Y to be non-zero. This means that the L' charge of ν_{2R} and/or ν_{3R} must also be non-zero and have opposite L' charge to ν_{1R}. For definiteness we assume ν_{2R} to have opposite L' charge to ν_{1R} which means that A and B are both zero and X can be non-zero. If the L' charge of ν_{3R} is non-zero, then $C = 0$ and either Y or Z is also zero which means that there is a zero eigenvalue. Thus, we must have the L' charge of ν_{3R} being zero. In other words, the requirement that all three bare right-handed neutrino eigenvalues are non vanishing together with the assumption that L' charge of at least one of the right-handed neutrinos is non-zero leads uniquely to a right-handed Majorana mass matrix of the form:

$$(\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R}) \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & C \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix} \tag{6}$$

Not only is the form of the right-handed mass matrix unique but we must have

$$\alpha = -\beta, \ \gamma = 0. \tag{7}$$

(Obviously this is only unique up to trivial permutations of the ν_{iR} and α, β, γ).

For quantum consistency, all gauge anomalies involving $U(1)_{L'}$ must vanish. The anomaly cancellation[5, 6] conditions are:

$$SU(2)_L^2 U(1)_{L'} \ \Rightarrow \ a + b + c + 3\epsilon = 0 \tag{8}$$

$$U(1)_{L'}^3 \ \Rightarrow \ \alpha^3 + \beta^3 + \gamma^3 - a^3 - b^3 - c^3 = 0 \tag{9}$$

$$\text{mixed gauge} - \text{gravitational anomaly} \ \Rightarrow \ \alpha + \beta + \gamma - a - b - c = 0 \tag{10}$$

All other anomaly conditions do not give independent constraints.

Eq.(7,8,9,10) imply that $\epsilon = 0$, and either a, b or c is zero. Thus, we find that the most general anomaly free form for L', consistent with a non-vanishing right-handed neutrino bare eigenmasses, has the form:

$$L' = c_1 (L_{\nu_{1R}} - L_{\nu_{2R}}) + c_2 (L_e - L_\mu) + c_3 Y \tag{11}$$

where c_1, c_2 and c_3 are arbitary numbers. Obviously the form is unique only up to permutations of ν_{iR} and $L_{e,\mu,\tau}$.

So far we have not discussed how the extended gauge symmetry is broken. We first examine the simplest case of just one exotic Higgs multiplet, h. The most natural scale for the symmetry breaking scale is $\langle h \rangle \lesssim TeV$ to avoid the gauge heirarchy problem. However, $\langle h \rangle$ cannot be too low, otherwise phenomenological problems will arise. In particular, if h also breaks electroweak symmetry, then this would make the Z' light ($M_{Z'} \lesssim M_Z$) which would be difficult to reconcile with existing experiments. It is natural, therefore, to assume that h is an electroweak singlet.[1]

[1] Of course, this need only be the case if there is one exotic scalar field. Later we will briefly consider next to minimal models, with two exotic Higgs multiplets, one of which can be an electroweak doublet. This doesn't cause phenomenological problems if the vacuum expectation value (VEV) of the electroweak singlet Higgs dominates over the electroweak doublet VEV's.

This means that Dirac neutrino masses, necessary for the see-saw mechanism to exist, must arise through coupling with the standard model Higgs doublet, ϕ:

$$\mathcal{L} = \lambda_{ij}\bar{f}_{iL}\phi\nu_{jR} \tag{12}$$

This lagrangian is only L' gauge invariant (with at least two non-zero $\lambda' s$) if $|c_1| = |c_2|$ (and c_1 can be fixed to 1 without loss of generality). This means that L' can be taken as

$$L' = L_{\nu_{1R}} - L_{\nu_{2R}} + L_e - L_\mu + \omega Y \tag{13}$$

or any of the other two physically distinct permutations[2]. Note that in the above basis, both the Dirac neutrino mass matrix and charged lepton mass matrix are both necessarily diagonal.

Since $L_e - L_\mu$ is a symmetry of the mass matrix it follows that the effective Majorana left-handed neutrino mass matrix has the form:

$$(\bar{\nu}_{1L}, \bar{\nu}_{2L}, \bar{\nu}_{3L}) \begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix} \begin{pmatrix} (\nu_{1L})^c \\ (\nu_{2L})^c \\ (\nu_{3L})^c \end{pmatrix} \tag{14}$$

This is the result in the absence of any coupling of the exotic Higgs h to the neutrino mass matrix. We see that two flavours are maximally mixed but degenerate. It is natural to assume that the exotic Higgs h couples to fermions (so that its gauge quantum numbers can be uniquely defined, c.f. ref.[8]), which will induce corrections to the neutrino mass matrix. There are just two possibilities (assuming h is an electroweak singlet), corresponding to h having gauge quantum numbers, $h \sim (1, 1, 0, +1)$ or $h \sim (1, 1, 0, +2)$ under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{L'}$. In the first case the right-handed Majorana mass matrix has additional terms coming from

$$\mathcal{L} = \lambda\bar{\nu}_{1R}h(\nu_{3R})^c + \lambda'\bar{\nu}_{2R}h^*(\nu_{3R})^c + H.c. \tag{15}$$

and so the right-handed Majorana mass matrix becomes:

$$(\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R}) \begin{pmatrix} 0 & X & \epsilon_1 \\ X & 0 & \epsilon_2 \\ \epsilon_1 & \epsilon_2 & C \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix} \tag{16}$$

where $\epsilon_1 = \lambda\langle h\rangle, \ \epsilon_2 = \lambda'\langle h\rangle$.

In the second case the right-handed Majorana mass matrix has additional terms coming from

$$\mathcal{L} = \lambda\bar{\nu}_{1R}h(\nu_{1R})^c + \lambda'\bar{\nu}_{2R}h^*(\nu_{2R})^c + H.c. \tag{17}$$

and the right-handed neutrino mass matrix has the form:

$$(\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R}) \begin{pmatrix} \epsilon_1 & X & 0 \\ X & \epsilon_2 & 0 \\ 0 & 0 & C \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix} \tag{18}$$

The second case features large angle oscillations between two of the flavours with the other flavour completely decoupled, while in the first case, we will have large angle oscillations between 2 flavours with the other two angles being small. Neither of these possibilities is compatible with the currently popular 3-flavour solution to the neutrino anomalies. If the popular neutrino solution pans-out then either one of our three basic assumptions is incorrect, or symmetry breaking is non-minimal (an example of non-minimal symmetry breaking will be given later-on).

The first case might possibly be consistent with experimental data if light sterile neutrinos exist. In particular, the atmospheric neutrino anomaly could be due to $\nu_\mu \to \nu_s$ oscillations[9][3] with solar oscillations being due to $\nu_e \to \nu_\tau$ oscillations. This scenario is potentially compatible with the LSND experiment[12], although the latter experiment

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[2] The phenomenological implications of models with gauged lepton number differences has been discussed previously in Ref.[7].

[3] The superKamiokande collaboration have argued[10] that $\nu_\mu \to \nu_s$ oscillations are disfavoured relative to the $\nu_\mu \to \nu_\tau$ possibility. However, as emphasised in Ref.[11], the $\nu_\mu \to \nu_s$ hypothesis is still a possible solution to the atmospheric neutrino anomaly. Long baseline experiments will ultimately decide this issue.

requires confirmation. In order to incorporate the large angle $\nu_e \to \nu_\tau$ oscillations, this case would require the L' symmetry to be

$$L' = L_{\nu_{1R}} - L_{\nu_{3R}} + L_e - L_\tau + \omega Y \tag{19}$$

The neutral lepton mass matrix for ν_L, ν_R has the form:

$$\mathcal{L}_{eff} = \frac{1}{2}(\bar{\nu}_L \ \overline{(\nu_R)^c}) \begin{pmatrix} 0 & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + H.c. \tag{20}$$

In the see-saw limit where the eigenvalues of M_R are much larger than the eigenvalues of M_D, the right and left neutrino states are effectively decoupled:

$$\mathcal{L}^{see-saw} \simeq \frac{1}{2}\bar{\nu}_L M_L (\nu_L)^c + \frac{1}{2}\overline{(\nu_R)^c} M_R \nu_R \tag{21}$$

where

$$M_L \simeq -M_D M_R^{-1} (M_D)^\dagger \tag{22}$$

In the case of Eq.(19), M_R is given by:

$$M_R = \begin{pmatrix} 0 & \epsilon_2 & X \\ \epsilon_2 & C & \epsilon_1 \\ X & \epsilon_1 & 0 \end{pmatrix} \tag{23}$$

and M_D is diagonal:

$$M_D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{24}$$

Evaluating the effective light neutrino mass matrix, M_L, in the limit where $\epsilon_{1,2} \ll X, C$, we find it has the form:

$$M_L = \begin{pmatrix} 0 & \omega_2 & y \\ \omega_2 & x & \omega_1 \\ y & \omega_1 & 0 \end{pmatrix} \tag{25}$$

where

$$x \simeq m_2^2/C, \ y \simeq m_1 m_3/X, \ \omega_1 \simeq -m_2 m_3 \epsilon_2/XC, \ \omega_2 \simeq -m_1 m_2 \epsilon_1/XC. \tag{26}$$

Clearly we expect $\omega_i \ll x, y$ (note that the ω_i vanish in the limit that $\epsilon_i \to 0$). If the Dirac neutrino masses are heirarchial, it is natural to expect $\omega_2 \ll \omega_1$, and we will examine this limit for definiteness. This matrix corresponds to the following oscillation pattern:

$$\begin{array}{ll} \text{Approximately} & \text{maximal } \nu_e \leftrightarrow \nu_\tau \text{ oscillations with } \delta m^2 \simeq \dfrac{2xy\omega_1^2}{y^2 - x^2} \\ \text{Small angle} & \nu_e \leftrightarrow \nu_\mu \text{ oscillations with} \sin^2 2\theta_{e-\mu} = \dfrac{4\omega_1^2 y^2}{(x^2-y^2)^2}, \ \delta m^2 \simeq x^2 - y^2 \\ \text{Small angle} & \nu_\tau \leftrightarrow \nu_\mu \text{ oscillations with} \sin^2 2\theta_{\tau-\mu} = \dfrac{4\omega_1^2 x^2}{(x^2-y^2)^2}, \ \delta m^2 \simeq x^2 - y^2 \end{array} \tag{27}$$

If we apply the $\nu_e \leftrightarrow \nu_\tau$ oscillations to the solar neutrino problem and the $\nu_e \leftrightarrow \nu_\mu$ oscillations to explain the LSND data, then we require:

$$\begin{aligned} y &\approx \sqrt{|\delta m^2_{lsnd}|} \approx 0.5 - 1eV \\ x &\approx \frac{2y}{\sin^2 2\theta_{lsnd}} \frac{\delta m^2_{solar}}{\delta m^2_{lsnd}} \sim 10^{-1}\ eV \\ \omega_1 &\sim 10^{-2}\ eV \end{aligned} \tag{28}$$

There are two obvious problems with the above interpretation of the solar and LSND data. First, the $\nu_e \leftrightarrow \nu_\tau$ oscillations are predicted to be approximately maximal, which gives a poor fit to the solar data: the hypothesis of maximal oscillations is allowed by the SNO data at only about 1% C.L. ($\chi^2/d.o.f = 55.3/34$. [13]). Second, the required value of ω_1 is uncomfortably large (or equivalently, this scheme would suggest a value of $\sin^2 2\theta_{e-\mu} \ll 10^{-3}$, the value favoured by the LSND experiment). Neither of these problems constitutes a rigourous experimental exclusion of this scheme. Future data may well exclude it but for now it is possible (but disfavoured by the data).

To explain the atmospheric neutrino data, one can assume the existence of at least one light sterile neutrino maximally mixed with ν_μ as in Ref.[14]. This comes about naturally if a mirror sector exists[15, 16]. Only $\nu_{\mu L}, \nu_{2R}$ couple to their mirror partners because only these particles have zero L' charge. This leads to an effective mass matrix for the 3 light ordinary and 3 light mirror neutrinos of the form:

$$M_L = \begin{pmatrix} 0 & 0 & y & 0 & 0 & 0 \\ 0 & x & \omega_1 & 0 & \delta & 0 \\ y & \omega_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & y \\ 0 & \delta & 0 & 0 & x & \omega_1 \\ 0 & 0 & 0 & y & \omega_1 & 0 \end{pmatrix} \tag{29}$$

where the δ term is the effective $\bar{\nu}_{\mu L}(\nu_{\mu L})^c$ mass mixing caused by the coupling of the ordinary and mirror sectors. The effect of the δ is to cause maximal oscillations between the ordinary and mirror neutrinos with the largest δm^2 occuring for $\nu_\mu \leftrightarrow \nu'_\mu$ oscillations. In fact we find $\delta m^2_{atm} \simeq 4x\delta$. The maximal oscillations between $\nu_e \leftrightarrow \nu'_e$ and $\nu_\tau \leftrightarrow \nu'_\tau$ are governed by, $\delta m^2_{11}, \delta m^2_{33}$ satisfying:

$$\delta m^2_{11} \;\approx\; \delta m^2_{33} \approx \frac{2\omega_1^2 \delta}{y} \tag{30}$$

If the $\nu_e \leftrightarrow \nu_\mu$ oscillation interpretation of the LSND experiment is correct, then

$$\delta m^2_{11} \approx \sin^4\theta_{lsnd} \frac{\delta m^2_{lsnd}}{16} \frac{\delta m^2_{atm}}{\delta m^2_{solar}} \sim 10^{-5}\ eV^2 \tag{31}$$

This would imply a much larger solar boron neutrino flux then predicted by standard solar models (c.f. Ref.[17]). Alternatively, if the LSND experiment is not confirmed then we can have $\sin^2 2\theta_{e-\mu} \ll 10^{-3}$ and $\delta m^2_{11} \approx \delta m^2_{33} \lesssim 10^{-11}\ eV^2$, implying that the $\nu_e \leftrightarrow \nu'_e$ and $\nu_\tau \leftrightarrow \nu'_\tau$ oscillations would have no effect for the solar neutrino experiments.

So far, we have focussed on the minimal see-saw model without gauge heirarchy problem. The minimal model has the theoretical advantage of making definite predictions for the structure of the neutrino mass matrix. The disadvantage is that the minimal model is experimentally disfavoured by a variety of experiments, although it is not yet definitely ruled out. If future experiments rule out the minimal model (by e.g. confirming the $\nu_\mu \to \nu_\tau$ oscillation interpretation of the atmospheric neutrino anomaly), then it means that at least two exotic Higgs multiplets are required. Unfortunately, this makes possible scenarios much more (theoretically) arbitrary. For illustration, we give one relatively simple model, which we now outline.

We assume that the gauge symmetry contains $L' = L_e - L_\mu + L_{\nu_{1R}} - L_{\nu_{2R}}$, which is broken by two exotic Higgs multiplets h_1 and h_2. The h_1 is an electroweak singlet, with VEV $\sim$ TeV. The h_1 may couple to the right-handed neutrino sector, either by Eq.(15) or Eq.(17). The h_2 is an electroweak doublet, $h_2 \sim (1, 2, +1, +1)$ under the gauge group, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{L'}$. This means that h_2 couples in the following way to leptons:

$$\mathcal{L} = \lambda_1 \bar{f}_{1L} h_2 e_{3R} + \lambda_2 \bar{f}_{3L} h_2 e_{2R} + \lambda_3 \bar{f}_{2L} h_2^c \nu_{3R} + \lambda_4 \bar{f}_{3L} h_2^c \nu_{1R} + H.c. \tag{32}$$

In the model where h_1 couples via Eq.(15), so that $h_1 \sim (1, 1, 0, +1)$, the Higgs field h_2 can gain a naturally small VEV, via terms such as $m h_1^c \phi^\dagger h_2$ in the Higgs potential. Anyway, the point of h_2 is that the Dirac mass matrix for the charged leptons and neutrinos is no longer necessarily diagonal. The charged lepton mass matrix has the form:

$$(\bar{e}'_{1L}, \bar{e}'_{2L}, \bar{e}'_{3L}) \begin{pmatrix} m_1 & 0 & a \\ 0 & m_2 & 0 \\ 0 & b & m_3 \end{pmatrix} \begin{pmatrix} e'_{1R} \\ e'_{2R} \\ e'_{3R} \end{pmatrix} \tag{33}$$

where $a = \lambda_1 \langle h_2 \rangle,\ b = \lambda_2 \langle h_2 \rangle$. This mass matrix can accomodate maximal mixing in the left-handed 2-3 block in the limit $a \to 0,\ |m_2 b| \gg |m_3^2 + b^2 - m_2^2|$. If we assume that $\lambda_{3,4}$ are small so that the neutrino mass matrix

is diagonalized by approxiately maximal mixing in the 1-2 sector, then the effective light neutrino mixing matrix becomes approximately:

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{34}$$

That is, bimaximal mixing results[18]. Of course, course this is not uniquely predicted and deviations can easily occur (and are suggested by the data). In fact, any solar angle can be accomodated if $\lambda_{3,4}$ are non-zero. Certainly, it would be interesting to do a detailed systematic study of next to minimal models, something we leave for the future[4].

To conclude this work, we have shown that the see-saw neutrino mass mechanism can coexist naturally with an extended gauge symmetry, without any gauge heirarchy problem, provided that the gauge symmetry contains gauged lepton number differences. The minimal model of this type was examined, which was shown to give definite predictions for the neutrino oscillation pattern which is theoretically very interesting despite the relatively poor agreement of the derived model with the current experimental data. We have also shown that the next to minimal models can accomodate a wider spectrum of oscillation patterns, including approximately bimaximal neutrino oscillations.

Acknowledgements The author would like to thank R. R. Volkas for his comments on a draft of this paper and for checking some of the equations. Comments by K. McDonald are also greatfully acknowledged. This work was supported by the Australian Research Council.

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[4] One interesting feature of next to minimal models is that we can embed the $U(1)_{L'}$ into a simple group such as $SU(2)$. That is, we have gauge symmetry: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)$. [This is not possible in the minimal model with only one exotic Higgs multiplet because they have phenomenologically unacceptable mass relations for the charged leptons]. Embedding the $U(1)_{L'}$ into simple groups such as $SU(2)$ may lead to more predictive and therefore more interesting models.